10/2/21

EC 2213 - PLANETARY CLIMATE

Climate - weather conditions prevailing in an area in general or over a long period.

Consider the graph of daily temperature in Machilipatnam (East coast) in May 2000
- Temperature is very variable - no two days are the same
- There's a diurnal cycle : cool → hot → cool based on time of day
- There was a sudden rise in T around mid-May, where it went till 45°C. perhaps indicating heat wave

Consider the annual temp gramph -
- Nov - Feb is cold and May is exceptionally warm
- Daily variability disappears when you zoom out
- There are some spikes & dips that stand out
- Greatest variability is between June - Sept

Consider the decade graph (2000 - 2010) -
- Annual pattern of varying temperature based on months
- Some years are hotter/colder than others.

Its reassuring to see some pattern in the highly variable data. Is this true for other variables too?

Station level pressure (hPa) - measured at 2m above the ground

Atm pressure is the result of the air column above (directly) that place.

The graph of this over a decade also shows a pattern.

Note that there's a 180° phase shift i.e high T → low p.

Thus we define climate as the statistically steady state. i.e. it shows an average pattern or regularity.

There is departure from the average - ∃ variability at all time scales. Variability in < year is not considered as climate variability. Only variation over 2 yrs or more is considered.

②

In the surface pressure graph, notice an unusually low surf. P reading – lowest in the decade – around the end of June in 2007.

This was due to Cyclone Yemyin (June 22)
The centre of a cyclone has very low pressure. Yemyin made landfall near Machilipatnam, so the S.P of the area dropped anamolously.
This is <u>not</u> climate variability.

Important to keep the context of climate study in mind –
- Region : we'll focus on planetary
- Time scale : years, decades, centuries, holocene etc
- Quantity : rainfall, temperature

Climate variability : What causes this?
How it is caused?
On what time scales?

Studying climate and its variability –
- ★ the physical principles that underlie it are surprisingly simple & elementary : conservatⁿ of energy, mass, momentum
- ★ These principles are embedded in very ~~a~~ complex set of interactions, which makes the study very complex
Hard to predict because of emergence.

"Climate science is to think deeply about simple things"

18/2/21

Lecture 2 (Intro)

Applied Math Methods
The model we're trying to build should be based on the physical phenomena we're studying. Geoscience –
- Deals with birth, life and death of planets
- Generously borrows from physics, chem & math but it more than just that. Hence, it needs its own vocabulary
- has phenomenan that occur over huge range of scales – length : AU to Å , time : μs to giga years.

Place of Geoscience –

- Cosmology : Birth, life and death of the universe
- Astronomy & astrophysics : nebulae, galaxies, planetary systems, stars their composition, associated processes
- Geosciences : what remains – a lot remains

For example –

* • Earth is made of inner core (solid), outer core (liquid), mantle, asthenosphere, crust (hydrosphere, atmosphere, cryosphere)
  • The inner core is made of solid iron & the mantle is responsible for transferring energy from inner core to the crust. (surface of the planet)
* Dynamics of crust & surface – tectonics, geomorphology
  The crust is floating on the mantle. Geosciences studies the formation and subduction of crust. This is important to study the configuration of continents, boundaries of ocean etc – this also affects the climate
* Dynamics on the surface – rivers, glaciers, weathering
  Features formed because of glaciers and evolution of surface features
* Hydrosphere and hydrology – water reservoirs & water cycle
* Atmosphere – study of winds, rainfall, weather
  Study of different layers of atmosphere and processes that occur in them.
* Biosphere : biogeochemistry, ecosystems, evolution.

All of this is studied through an analytical lens

Objects of study – Matter
Energy
Sources & Sinks (of matter & energy)
How does it move from source to sink?
How quickly? How much?

(1)

Lecture 2 - BIG QUESTIONS

17/2

Ahistorical sciences - natural sciences like physics, chem

Historical sciences - geology, climate science, biology, history

These sciences are sensitive to time - they talk about evolution of systems over time.

The main aim here is to stitch a story from the data available which is within the constraints of physical laws.

Formation of Crust

- Earth formed ~ 4.6 billion years ago from nebula of a dead star
- The process of accretion of matter towards gravitational center formed magma which had started cooling
- A planetesimal, Theia, hit earth and whatever crust had formed melted again & this collision resulted in moon
- At this point, the surface of earth was magma ocean. It released volatile compounds which were retained as the atmosphere
- Volcanoes - sudden release of magma and volatile compounds. Its a pathway from interior

How long did it take for magma to cool and form the crust? How did it lose heat? Dependence on atmospheric composition?

Once the crust was formed, it acts as an insulating layer - energy from centre of earth to the surface was cut off.

- Faint Young Sun Paradox.

Every star goes through a life cycle where its luminosity increases over time

So, how did the evolution of this sun affect the climate. We know that liquid water existed on earth ~ 4 billion years ago. How did we have water with a faint sun?

4bya, with Sun's output only 70% of current output, we'd expect the earth to be frozen but there was liquid water!

People hypothesize that atmospheric composition (H_2O, CO_2) was the reason for liquid water

Long term stability of surface temperature - once we've had liquid water, we've maintained it despite the increase in luminosity of the sun. How?

It was known that we had liquid water H_2O 4bya and the faint young sun by late 19th / early 20th century. It was only in 1972 that in a paper, Sagan and Mullen questioned how climactic stability was possible when the sun was getting hotter

Goldilocks zone - Habitability problem
How near / far away from a star does the planet need to be for it to sustain life?
In our solar system, Venus, Earth and Mars started out similarly but Earth was in a position to retain liquid water and atmosphere
The planet needs to maintain it for very long to allow complex life to evolve. This also depends on the age of the star.

Neoproterozoic Snowball Earth (~700 Mya)
There was a purely theoretical paper exploring the model of snowball earth. Later geologists started finding evidence for it - that ice covered portions near the equator.
Neoproterozoic era saw huge climate fluctuations like never before and never after & people are not sure why.
Questions - What caused it?
How did we get out of it?
What followed?

Questions closer to home -

- Eocene's equable climate (~50 Mya)
  It was a hothouse - crocodiles in Antarctica and palm trees near the arctic. The temperature gradient between equator and pole was much lesser than current value
  Its only very recently that models have been able to simulate these conditions.

- Stable Holocene (last 10k yrs)
  Evolution of complex human civilisations has been attributed to the stability of Holocene. Do we know what causes it and how to maintain it?

- Anthropocene (last 150 years)
  - Started when humans started altering the climate severely. There have been two instances where life significantly affected climate -
  - Great oxidation event : 4 Bya (Paleoproterozoic). Photosynthetic organisms changed the conc of O_2 in atmosphere altering the composition of water (oceans), crust & atmosphere
    Iron in the ocean precipitated & atmosphere was filled with a highly reactive gas.
  - Current Great Carbonification ? : Due to human activity there is an unprecedented increase in CO_2. Are we going to destabilize the relatively stable climate ?
    How are we going to do it? - Melting of Arctic sea ice ? Change in rainfall pattern?

Lecture 03

Energy Balance

A planet has huge volume and mass. But we are considering the crust, oceans and atmosphere
Also, not all planets have a solid surface i.e. they are gas or ice giants.
To understand the climate of exoplanet through limited observation, we need to have good understanding.

What are the variables of interest?

The main variable is temperature which directly correspond to energy balance. Other variables like wind, rainfall depend on T. Temp also helps us describe habitability.

Energy budget

How does a planet lose, gain ~~or~~ and use energy?

Energy sources

- Core

Lot of energy has been trapped inside earth through –

- gravitational potential energy from the collapse of materials that form earth
- Radioactive decay of elements trapped in core
- Friction due to tides induced by the moon. This happens because of the 'braking effect' of moon on earth. Its relatively lesser but when/if moon was closer to earth, then this source of heat would be non-negligible

The crust is a very good insulator. So the energy in the earth's interior that comes to the surface is negligible for a rocky planet in Earth's position.

- Sun

The major source of energy is the sun.

Energy sinks

* Losing it as Kinetic Energy by losing some material from planet such as atmosphere But big planets don't lose material
* Major energy sink is through radiation to space.

8

Radiative Balance

* We can study this by analysing data from earth. The radiation is recorded from the top of the atmosphere during March-April-May (Equinox).

* The equator is closest to the sun and that is reflected in the graph. Incoming solar radiation is least at the poles

* There is minimal variation in T along the latitude. This is because the earth rotates fast enough to negate such variation

There is a gradient from Eq to poles which is due to cosine ($\cos\theta$) of the angle b/w incoming radiation and the surface

Outgoing Longwave radiation

This is the energy radiated from earth. Here the range of radiation emmitted is much lesser (150 - 300) in comparison to the incoming radiation (0-400).

The desert regions and sub-tropical regions show higher radiation.

The annual mean pattern of outgoing radiation doesn't change much. So we say that earth as a whole radiates at some freq overall (the variation observed is much much lesser compared to that of sun).

Net Radiation

Certain regions close to equator show a net gain in energy and beyond 30°, there's a net loss in energy.

If there was no compensation mechanism, the pole would be completely frozen and equator would get hotter.

So there's some energy transfer from equator to pole.

Understanding different terms and questioning the facts in the diagram of Earth's energy budget.

Cone – vertex angle : $\Delta\theta$

3D angle – $2\pi(1-\cos\Delta\theta)$

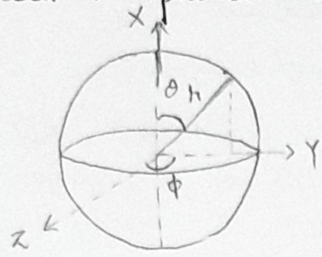
24/2/2021

Lecture 04

Blackbody Radiation

One of the main sources and sinks of energy of a planet is through Electromagnetic radiation.

Recall : Spherical coordinates



Along a circle centred on reference directⁿ

Azimuthal angle

ϕ : Angle between the vector & the horizontal

θ : Angle b/w vector & vertical line

$\theta \in [0, \pi]$ $\quad$ $\phi \in [0, 2\pi)$

(Ω) Solid angle : 1 steradian is the angle subtended by an area of r^2 on the surface of sphere.

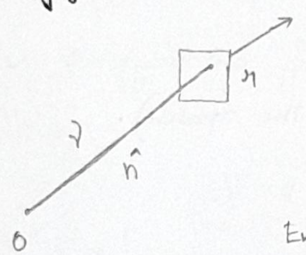
So, $\boxed{\Omega = \dfrac{dA}{r^2}}$ $\quad$ $dA = d\phi\, d\theta \sin\theta = -d\phi\, d(\cos\theta)$

$\boxed{d\Omega = -(d\cos\theta)\, d\phi}$

For a circle, $\Omega = 4\pi$

$$\Omega = -\int_1^{-1} d\cos\theta \cdot \int_0^{2\pi} d\phi = 4\pi$$

Energy and Irradiance



Consider a point at a distance r from the origin in direction $\hat{n}$

The radiation of frequency ν and spectrum 'sigma' (Σ) is incident on the point.

Energy at this point in the neighborhood is given by –

$$\Sigma\, dV\, d\nu\, d\Omega$$

dV : small volume around r

$d\nu$: takes care of ν dependency of spectrum

$d\Omega$: small solid angle that we consider.

If we consider an area perpendicular to the radiation. The total energy passing through the area is given by –

$c\,\Sigma\, dA\, d\nu\, d\Omega$: Flux of energy through Area dA

$c\Sigma$: $W m^{-2} Hz^{-1} sr^{-1}$: flux spectrum / Spectral irradiance

When flux spectrum is integrated over all frequency, it gives us irradiance.

Quantum of energy for EM radiat^n having frequency ν is - $\Delta E = h\nu$

Max planck computed the spectral irradiance for a black body by assuming that energy is quantised. Irradiance is given by -

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \cdot \cos\theta \quad — \quad (\star)$$

This is irrespective of $\hat{n}$ (direction of radiation) i.e its isotropic - flux spectrum is same no matter what direction you chose. i.e. its equally intense in all directions.

If radiation is not normal to the area,

flux : $c\,\Sigma\, d\nu\, d\Omega\, dV \cos\theta = B\, dA\, d\nu\, d\Omega \cos\theta$

If blackbody is kept at the centre of a sphere, then we can find the flux by simply integrating over all θ and ϕ -

Flux $= \pi B\, dA\, d\nu$

To know the flux of energy exiting the surface of a blackbody through a small, nearly flat patch with area dA over bandwidth $\delta\nu$ -

$\Rightarrow (B \cdot dA \cdot d\nu\, d\Omega) \cos\theta \quad — \quad (1)$

$d\Omega = 2\pi\, d\cos\theta$ (for all rays making an angle θ relative to the normal to the patch).

$B\cos\theta\, d\Omega$

Integrating $B\cos\theta\, d\Omega$ from $\theta = 0$ to $\theta = \pi/2$

$$B\cos\theta\, d\Omega = \int_1^0 B\cos\theta \cdot 2\pi \cdot d\cos\theta = B \cdot 2\pi \left.\frac{\cos^2\theta}{2}\right|_0^1 = B\pi$$

$\therefore$ Flux through patch is -

$$B \cdot dA\, d\nu \cdot d\Omega \cos\theta = \underline{\pi B \cdot dA \cdot d\nu}$$

Total irradiance,

Take $u = \frac{h\nu}{kT}$, so in eqn $(\star)$,

$$B(u, T) = k_1 \frac{u^3}{e^u - 1} \quad \text{where } k_1 : \text{some constant} = \frac{2k^3T^3}{h^2c^2}$$

$u \ll 1 \quad \because h\nu \ll kT$ then, $\frac{u^3}{e^u - 1} \rightarrow \sim u^2$

Substituting values back, $B \approx 2kT\nu^2/c^2$ independent of h !!!

But if $B \sim \frac{2KT\nu^2}{c^2}$, emmission would increase with frequency without bound $\Rightarrow$ (infinite energy)

But this contradicts with our observation

→ No matter what energy blackbody has, it can find a suitable frequency, emit energy and cool instantaneously.

But for $u > 1$, total energy emmitted ~~through~~ is finite because $\exp(u)$ decays faster than u^3 increases.

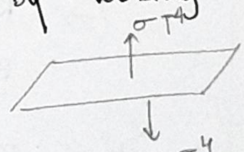
To find the total amount of energy emmitted, the irradiance is integrated over all frequency -

$$\int_{\nu} \pi B(u,T)\,.\,d\nu = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8}$$

Stefan-Boltzmann Law - gives energy emmitted by <u>unit area</u>

Also energy emmitted is just based on temperature of blackbody

Using this (& assuming stuff to be blackbody) we can infer properties about it by looking at the radiation emmitted



Electromagnetic spectrum -

Most of energy emmitted by -

planets (~ 100s K) - infrared spectrum (longwave)

stars (~ 1000s K) - visible spectrum (shortwave).

- When we compare theoretical spectrum to real world observational data, it is very accurate.

Accomplishment of physics!

- Spectrum of earth - doesn't conform as well. There are some 'windows' which actually make it easily habitable & gives leeway of composition.

(12) Lecture 05 6/8/21

Zero dimensional model

Its the most fundamental model of energy balance

Energy source and sink — radiation

To maintain a stable climate, the planet has to have constant total energy.

R_i : incoming radiation R_o : outgoing radiation

We only consider the surface — blackbody

→ But on earth, some energy received from the sun is reflected — αR_s

α : albedo R_s : solar radiation

$$\alpha^{-1} = \frac{\int R_i}{\int R_{ref}}$$ R_p : radiation reflected from planet

α is a integrated value, calculated over the spectrum

So we need to know these 3 values

R_s, R_p and α.

→ Sun's radiation is very close to a blackbody's

$\Rightarrow R_s \propto \sigma T_s^4$ $T_s \sim 6000\,K$ $R_s = k\sigma T_s^4$

depends on distance b/w earth & sun

$R_p \doteq \sigma T_e^4$

In the Z-D model, the planet is basically a point with albedo α and temperature T_e

→ Questions : $T_e = ?$ $\propto \alpha$? proportional to albedo

We crudely assume that, $T_e = T_{surf}$ $\alpha = \alpha_{surf}$

S : solar constant $= R_s = \sigma T_s^4$

$$\therefore \frac{S}{4}(1-\alpha_{surf}) = \sigma T_{surf}^4$$: Fundamental eq^n of climate

→ What can we gain from such a model?

no moderating mechanism For moon's local surface, this model gives a pretty good approximation. Since it has no atmosphere (or other energy transfer systems) the local temperature at any point follows the OD model. This predicts that ΔT between noon (350 K) and midnight (100 K)

So it does have practical use.

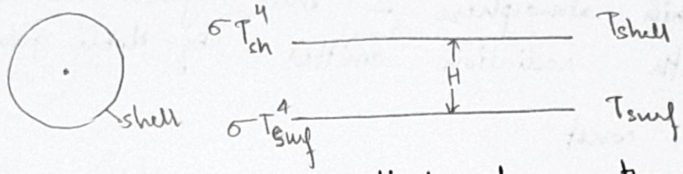
These are instantaneous values – not mean – which means the planet doesn't store the energy.

Because of these extremes, moon is not habitable

→ Shell model / $\frac{1}{2}$ dimensional model

* The planet is still a point, but its enveloped by a shell which absorbs part of R_s and has its own albedo.

So we've to consider two layers – T_{shell} & T_{surf}.

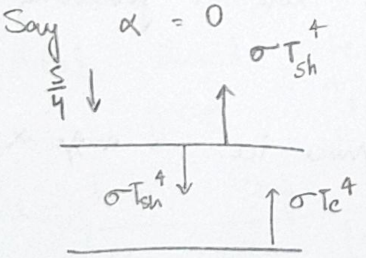


* The radiation is emitted from the shell – the energy balance is carried out wrt it

H : Radiating height

T_e : Radiating temperature (could be T_s or T_{sh})

Say $\alpha = 0$



$$\sigma T_{sh}^4 = \sigma T_e^4 \quad \text{i.e. } T_{sh} = T_e$$

$$\frac{S}{4} = \sigma T_{sh}^4$$

* Two Qs : How much is radiated?

From where is it radiated

Lecture 06

0D model : 2

Earth Day/Night Changes

There are very few places of earth where diurnal variability of T is more than ~ 20K.

One of the reasons — buffering capacity of oceans due to very high specific heat capacity of water.

This would be reasonable for planets — (assumptions)

- "Aqua" planet
- Uniform temperature — through energy transport from subsolar point to other regions
  ↳ if systems were vigorous enough
- Optically thin atmosphere — these gases don't interact with radiation emitted Eg. Noble gases, N_2

$\therefore$ $T_e \sim$ const

Aqua planet ⇒ possibility of ice
This makes a huge difference because of increase in albedo

$\alpha_{water} \simeq 0$ $\qquad$ $\alpha_{ice} \simeq 0.3 - 0.4$

Albedo has an important role in governing climate

Coupling : $\alpha_{surf} \propto T_{surf}$

i.e. lower T ⇒ more ice ⇒ more α_{surf}

More realistic case :

$T_{surf} = f(\phi)$: function of latitude

(14)

Lecture 06

0D model : 2

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More realistic case :

$T_{surf} = f(\phi)$: function of latitude

Lecture 07 — 6/3/21

Ice free and snowball states – Python tutorial

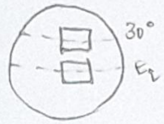
Neoproterozoic experienced several climate states – snowball vs hothouse states.

Variation of temperature with latitude –

$$T(\phi) = T_e - \Delta T \sin^2(\phi)$$

Functions – temp profile and weighted avg.

For averaging values on a sphere, it needs to be weighted by a cos function –



$$\frac{A_{Eq}}{A_{30^\circ}} = \frac{1}{\cos\phi}$$

Snowball state

Its integrated over all longitudes.

A latitude is considered ice free if ≥ 274 K and ice covered if $T < 274$ K.

To have a snowball state, $T_e < 274$ K.

What should be the ΔT for ice free state given T_e?

See how α_{avg} compares with the equation –

$$\boxed{\alpha = \alpha_i - (\alpha_i - \alpha_o) \cdot \frac{(T - T_i)^2}{(T_o - T_i)^2}} \quad \text{— (1)}$$

$\alpha_o = 0.2$

$\alpha_i = 0.5$

Lecture 08 – Snowball Earth Dynamics — 10/3

0D model becomes interesting because of non-linearity of –

- albedo wrt temperature
- radiation (OLR) wrt surface temperature

(16)

Simplest climate model that's realistic –

» OLRT (CO_2, T)

This is generated by fitting to a polynomial. In this case, it's $\log CO_2$ and some normalised temperature.

~~For~~ When OLR is plotted against T, its not $\propto T^4$, the graphs actually get flatter.

This is for when the earth's ~~T~~ is climate system is in equilibrium.

We can calculate avg. albedo using the equation (1).

There, T_p is dependent of T_e, ΔT and $\sin^2$

A quadratic fit seems to work well for albedo, but we'll see how models change when the albedo varies differently.

Snowball Earth model is an equilibrium case where net radiation is zero. i.e. input = output.

Its controlled by CO_2 and cloud forcing

net_rad^n (CO_2, T, cloud_forcing):

If CO_2 < 0,

return 1e^5

return solar_constant * (1 - albedo (T))/4 + cloud_forcing - OLR (CO_2, T)

This gives us net imbalance. We've to see how to take it to zero.

Constants –

T_ice_free = 280

T_ice_covered = 250

α_0 = 0.2

α_i = 0.65

solar_constant = 0.94 × 1367 → fainter sun

temp = range (200, 330)

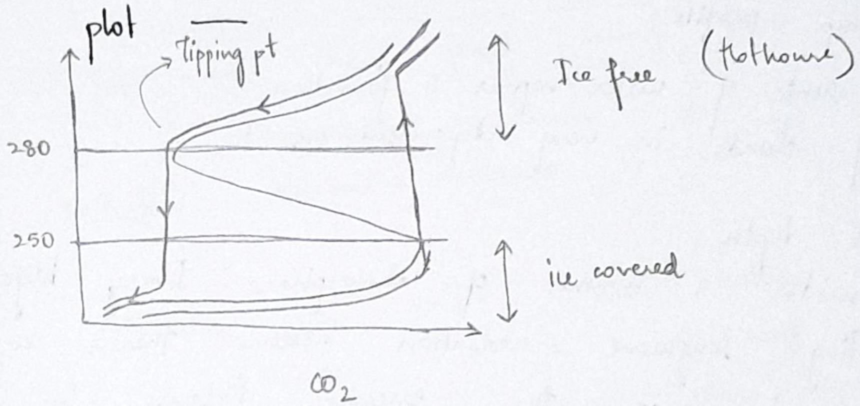
cloud_forcing = 0

guess = 1e-2

To find an equilibrium state, we've to find a solution for non-linear eq^n given by net radiation. This is done by the Newton-Raphson method, one among many.

Cloud forcing – albedo is set by clouds as well as the surface. i.e. clouds affect radiative balance in the shortwave and they absorb longwave band. So, it's complicated.
If reflective > absorptive, the forcing is said to be positive and vice-versa.

The plot



- Once we start decreasing CO_2, it turns from hothouse to an entirely ice-covered state. This is called the tipping point.
- When CO_2 is increased, it's still in ice-covered state until it suddenly becomes a hot house. No peaceful transition.
- To reach the middle part (partially ice covered state), we need to start out from intermediate conditions. But these states are unstable states whereas the other two arms are fairly stable.
- 1. Bifurcation : moving from unstable to stable state very quickly
  2. States – stable and unstable
  3. Hysteresis – From stable to unstable, it takes a lot of work

R/. Interpret this model.

PART 2

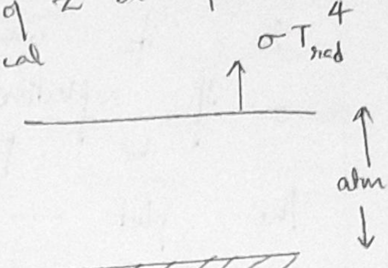
Lecture 9

Toward 1D Models — Model w/ continuous variation in absorption & emission & solving for thermodynamic equilibrium

Until now: Energy balance
Blackbody radiation
Shortwave: sun & albedo } Seperate spectra
Longwave: Radiating T }
0D Model
Radiating height

Everything relevant is a ~~model~~ function of z or any of its transform

Moving to 1D Planet ⇒ earth is symmetrical



Radiating T ?
Radiating height ?
Temp. profile ?
↳ amt of water vapour & formation of clouds is very dependent on T.

Optical depth

Consider a column of atmosphere. Every layer is emitting longwave radiation because there's a T profile. So what is the energy balance for one of the layers in the atmosphere?

Consider pressure as a vertical coordinate.

P_2 ——— ΔP
P_1 ———

Hydrostatic balance gives us —

$$m = \frac{\delta p}{g}$$

∵ pressure decreases with height & we want $\delta\tau_*$ positive

Kirchoff's law: κ = absorption coeff = emission coeff
κ: absorptivity / emissivity

Absorptivity / emissivity $= -\kappa \frac{\delta p}{g}$ → no. of particles

Emissivity: $\boxed{\delta\tau_* = -\kappa \frac{\delta p}{g}}$

of a layer is given by optical thickness ($d\tau_*$) — proportion of intensity absorbed in this layer

Absorption coefficient: κ — if $\frac{\kappa \Delta P}{g} > 1$, the slab acts like a blackbody

When $\frac{\kappa \Delta P}{g} < 1$ ⇒ Optically thin ⇒ IR can directly escape to space

Intensity of radiation at height z — $I(p, \hat{n}, \nu)$

flux density SPECTRAL IRRADIANCE

15/4

$\frac{d\tau_*}{dp} = \frac{-\kappa}{g}$ τ_* increases as P increases ?

P decreases with altitude, τ_* increases with altitude

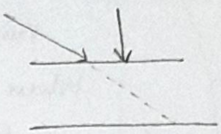
OPTICAL THICKNESS COORDINATE

We consider τ_* to be a kind of thickness

That's based on optical property

If the layer is transparent, optical thickness will be very less

κ itself can be a function of P, T



If radiation is incident normally, then $d\tau_*$ is the optical depth

But if its incident at an angle, it will encounters more molecules

⇒ It can be fixed using $\cos\theta$: $\frac{d\tau_*}{\cos\theta} = d\tau$

$d\tau$ is a convenient coordinate to figure out radiative balance because it directly depends on $d\tau$

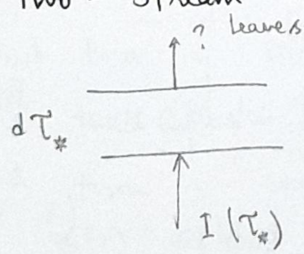
κ — depends on ν, P, T and nature of gas

$\kappa = \sum_i \kappa_i \frac{m_i}{\sum m_i}$ κ_i — Abs coeff of each gas

$\frac{m_i}{\sum m_i}$ — mass specific conc

IR radiation incident on each layer from below — NOT above

Two - Stream Equations



Absorbed $\propto I(\tau_*) \cdot \frac{d\tau_*}{\cos\theta}$

Incident amt & absorptivity

Emissivity $= d\tau = \frac{d\tau_*}{\cos\theta}$

B : Black body radiation at any given frequency — $B \cdot d\tau$

Radiation emitted

Assume : no scattering

We can replace $I(p, \hat{n})$ with $I(\tau^*, \theta)$

When $\kappa \frac{\Delta P}{g} \gg 1$, its cut up into slabs where $= 1$. These layers act as blackbodies ⇒ perfect absorbers + emitters

Only the topmost layer will radiate IR to space — OLR depends on T of this slab only

$T_S = \left(\frac{P_S}{P_{rad}}\right)^{R/C_p} T_{rad}$

(20)

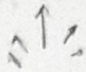
The radiation that comes out the top of the layer, is incident flux minus amount absorbed plus the small amount emitted –

Schwarzschild equation – GENERAL EQN FOR DISCRETE LAYERS

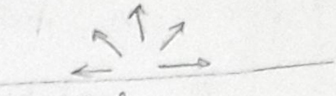
$$\boxed{I'_{(\tau_* + d\tau_*)} = \left(1 - \frac{d\tau_*}{\cos\theta}\right) I(\tau_*) + B \cdot \frac{d\tau_*}{\cos\theta}}$$

When incident radiation is normal, $\cos\theta = 1 \Rightarrow$ amount absorbed is minimal

Peaked



When $\theta = \pi/2$, practically the whole radiation is absorbed



So most of outcoming radiation is peaked at normal angles.

B_{ST} is isotropic in every layer

We'll see that $I(\tau_* + d\tau_*)$ is also isotropic simplest assumption.

Integrate I over all θ & ϕ limit $d\tau_* \to 0$

$$\frac{d}{d\tau_*} I(\tau_*) = \frac{-1}{\cos\theta}\left[I(\tau_*)_0 - B_{(T)}\right] \quad - (1)$$

We can derive an eq^n for I_+ : net upward flux per unit freq.

by multiplying (1) by $\cos\theta$ and integrating over all solid angle

RHS of Eq^n (1) is equivalent to –

$$\int I(\tau_*)\, d\Omega = \int_0^{\pi/2}\int_0^{2\pi} I(\tau_*) \cdot d\phi \sin\theta\, d\theta \quad \text{[Refer Pg 9]}$$

$$= \int_0^{\pi/2} I(\tau_*)\, 2\pi \cdot \sin\theta\, d\theta \quad (I \text{ is isotropic})$$

$$2 I_+ = 2\pi I \int_0^{\pi/2} \sin\theta \cdot d\theta \quad \Rightarrow I_+ = \pi I \quad (?) \checkmark$$

$\Rightarrow$ Decay rate is same as for unidirectional radiation propogating at an angle s.t. $\cos\bar{\theta} = \frac{1}{2}$ ie $\bar{\theta} = 60°$

We can allow for downward flux by allowing θ values between $\pi/2$ & π

$d\tau$ – amazing vertical coordinate. We don't have to worry about absorptivity / emissivity – all layers have same value! (21)

$$d\tau_v = \frac{d\tau_*}{\cos\bar{\theta}}$$

In terms of $d\tau_v$, equations for upward & downward flux are –

$$\frac{d}{d\tau_v} I_+ = -I_+ + \pi B$$

vertical coordinate : optical thickness not pressure.

$$\frac{d}{d\tau_v} I_- = I_- - \pi B$$

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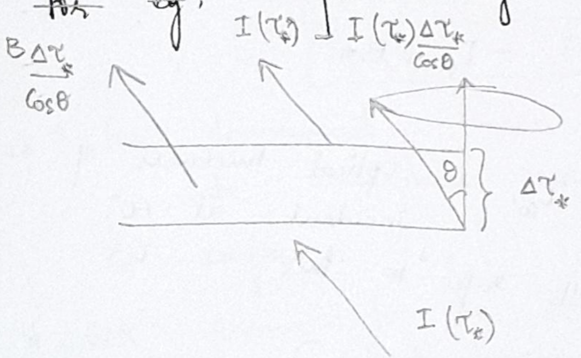
Lecture 10

Atmospheric radiative transfer

If absorption = 0 $\Rightarrow$ $d\tau = 0$

~~For~~ eg ; ~~experimentally~~.



Schwarzchild eq^n :

For given frequency,

$$I(\tau_* + d\tau, \theta) = \left(1 - \frac{d\tau_*}{\cos\theta}\right) I(\tau_*, \theta) + B(T(\tau_*)) . \frac{d\tau_*}{\cos\theta}$$

First term of RHS is transmitted flux – it's the the flux from previous layer $\times$ (1 – absorbance)

2nd term : emitted flux – determined by its temp.

B(T) : black body radiation produced at any temp T

B is isotropic. We multiply this by the gas' emissivity.

We're ignoring scattering.

In eq^n 1, RHS is simply B – I i.e total emitted – absorbed flux (scaled) gives us change in flux at any angle

(22)

a) If I is passing through a hemisphere, then the vertically upward projection of flux will be πI (~ project hemisphere on disk).

This is upward flux: $I_+ = \pi I$

b) Multiply I by $\cos\theta$ and integrate over the hemisphere – like taking all vertical components & adding them – will also give I_+.

Substituting this into Schwarzchild,

$$\frac{d(I)}{d\tau_*} = \frac{1}{\cos\theta}\left[-I(\tau_*) + B(T(\tau_*))\right]$$

Remove θ dependence

$$\frac{d}{d\tau_*}\int_{\substack{\text{upper}\\\text{hemisphere}}} I\cdot\cos\theta\cdot d\Omega = \int_{\substack{\text{upper}\\\text{hemisphere}}} (-I + B)\, d\Omega$$

$d\Omega$: solid angle

$$\frac{dI_+}{d\tau_*} = -2\pi I + 2\pi B = -2\cancel{\pi}\left(\frac{I_+}{\cancel{\pi}}\right) + 2\pi B$$

$$\Rightarrow \underbrace{\frac{1}{2}\cdot\frac{d}{d\tau_*}}_{d(2\tau_*)}(I_+) = -I_+ + B\pi$$

$2\tau_* = (\sec 60^\circ)\tau_* = \tau_{60^\circ}$: optical thickness of radiation incident at 60°.

We'll refer to this as τ

TWO - STREAM EQUATIONS

$$\Rightarrow \left\{\begin{array}{l} \dfrac{d I_+}{d\tau} = -I_+(\tau) + \pi B(T(\tau)) \\[2ex] \dfrac{d I_-}{d\tau} = I_-(\tau) - \pi B(T(\tau)) \end{array}\right.$$

: Upward flux

: Downward flux

Linear 1st ODE – they can be solved using integrating factors and using fluxes at the ground and sky to get a closed-form equation of flux at any τ.

Sol^n can be obtained by substituting $I_+ = A(\tau_\nu)e^{-\tau_\nu}$ which reduces the problem to evaluation of a definite integral for A.

used to weight the radiation ← Transmission function

The solutions are –

$$I_+(\tau) = I_+(0)e^{-\tau} + \int_0^{\tau} \pi B(T(\tau')) \cdot \underbrace{e^{-(\tau-\tau')}}_{} \cdot d\tau'$$

$$I_-(\tau) = I_-(\tau_\infty)\, e^{-(\tau_\infty-\tau)} + \int_{\tau}^{\tau_\infty} \pi B(T(\tau')) \cdot e^{-(\tau'-\tau)} \cdot d\tau'$$

1st term : boundary flux i.e ground or sky depending on upward or downward

It decays away from 0 to the optical height we're at.

2nd term : flux from each layer τ', given by πB, and decaying away exponentially too.

Its integrated over fluxes from all layers.

Boundary conditions of 2-stream eqns –

* ν :– $\boxed{I_+(0) = e.B(T_g)}$ e: emissivity

ν :– $\boxed{I_+(\tau_\infty) = \text{OLR}}$

* For downward fluxes, there's no significant influx into atmosphere $\Rightarrow$ $\underline{I_-(\tau_\infty) = 0}$

At surface : $\underline{I_{-,\nu}(0)}$ is the back-radiation.

* OLR grows to be dominated by emission of upper optical layers (physically thicker than lower ones) ↑ when $\tau \gg 1$

Similarly, back-radiation is dominated by contribution of lower optical layers.

Exponential attenuation of radiation ⇉ Special case #1 : Beer's law

or too cold to radiate

Consider a freq. in atmospheric window $\Rightarrow$ atmosphere doesn't radiate anything i.e $B_\nu(T(\tau)) = 0$
internal source vanishes $0 < \tau < \tau_\infty$

$\therefore \nu_w$: $I_+(\tau_\infty) = I_+(0)e^{-\tau}$ where ν_w is a freq in the window

$I_-(0) = I_-(\tau_\infty).e^{-(\tau_\infty-\tau)}$

(24)

→ Special case #2 : Infinite Isothermal slab

Isothermal medium ⇒ no boundary fluxes $(I_+(0) = I_-(\infty))$

each layer contributes a flux $\pi B(\tau, \nu)$ in both directions. Thus, $\pi B = I_+ = I_-$ at all heights and all frequencies

→ Special case #3 : Finite isothermal slab

Let the slab have an optical thickness τ_∞

Shifting the vertical coordinate : centre & will represent $\tau = 0$ and boundaries $\tau = \pm\tau_\infty$ respectively

Boundary fluxes ; assume boundary influxes are 0

i.e. $\underset{\text{base of slab}}{I_+\left(-\frac{\tau_\infty}{2}\right)} = \underset{\text{top of slab}}{I_-\left(\frac{\tau_\infty}{2}\right)} = 0$

This means that slab is going to cool as it's only losing heat

Since it's isothermal, we can take $\pi B(\tau, \nu)$ out.

Boundary term = 0. Integrating the exponential, we get –

$$I_+(\tau, \nu) = \left[1 - e^{-\left(\tau + \frac{\tau_\infty}{2}\right)}\right] . \pi B(\tau, \nu)$$

$$I_-(\tau, \nu) = \left[1 - e^{\left(\tau - \frac{\tau_\infty}{2}\right)}\right] . \pi B(\tau, \nu)$$

So radiation emitted at top of layer is $I_+\left(\frac{\tau_\infty}{2}\right) = \left(1 - e^{-\tau_\infty}\right)\pi B$

The exp → 1 ∵ $\tau_\infty \gg 1$

* So in an optically thick limit, $I_+ = I_- = \pi B$ through most of the layer

* In an extremely thin slab (τ, τ_∞ are small), using Taylor expansion, $I_+\left(\frac{\tau_\infty}{2}\right) = I_-\left(-\frac{\tau_\infty}{2}\right) = \tau_\infty \pi B$

Here, τ_∞ is the bulk emissivity of the layer

* Optically thick finite isothermal slab behaves like a black body with outer layers undergoing strong cooling – good model for certain clouds.

Heat is lost from inner layers as well!

$$h_\nu = -\left(e^{-\tau} + e^{\tau}\right) e^{-\tau_\infty/2}\, \pi B$$

Heating rate: Taking derivative of net flux wrt τ gives the difference b/w energy entering & leaving a thin layer

Upward flux $(I_+(\tau))$ contributes small amount of irradiance $-dI_+(\tau)$ — this is the energy attenuated while passing through the layer τ.

Similarly $I_-(\tau)$ contribute dI_-

Thus heating rate per unit optical thickness per unit freq. is given by –

$$h_\nu = \frac{d}{d\tau}\left(-I_+(\tau,\nu) + I_-(\tau,\nu)\right)$$

Converting to pressure coordinate allows us to divide this by specific heating capacity c_p -

Heating rates will also need to be integrated across all frequencies

Non-isothermal cases

Here, transmission fn $e^{-(\tau-\tau')}$ is too small & can be dropped

1. Optically thick limit

T varies continuously. $\Rightarrow d\tau \gg dp$ i.e thick enough to witness real change in temp (greater than fluctuations). We'll approximate this layer to be optically active part of environment — p_{ground} to $p(\tau_\infty)$

$\therefore \tau_\infty \gg 1$. [This maybe true for some freq and not others]

$$\nu :- \quad I_+(\tau) = I_+(0)e^{-\tau} + \int_0^{\tau} \pi B(T(\tau')).\, e^{-(\tau-\tau')} . d\tau'$$

$$\Rightarrow I_+(\tau) = 0 + \int_0^{\tau} \pi B(T(\tau')).\, e^{-(\tau-\tau')} d\tau'$$

atmosphere is optically thick $\Rightarrow$ influx = ?

$$I_+(\tau) = \int_{e^{\tau}}^{1} \pi B(T(\tau')).\, dt \quad \text{where } t = e^{-}$$

(26)

$$I_+(\tau) = \Big[\pi B(T(\tau')).\, t \;-\; \pi \int t \cdot \frac{dB}{dT}\cdot\frac{dT}{d\tau'}\cdot d\tau'\Big]_0^{\tau} \qquad t = e^{\prime}$$

Integrating by parts $\int u\,dv = uv - \int v\,du$

$$\simeq \pi\left[B(T(\tau)).e^0 - B(T_{surf})e^{-\tau} - \frac{dB}{dT}\Big|_{T(\tau)}\cdot\frac{dT}{d\tau}\right]$$

$$= \pi B(T(\tau)) - \pi \cdot \frac{dB}{dT}\Big|_{T(\tau)}\frac{dT}{d\tau}$$

$d\tau \rightarrow dp$ $\frac{g \cos\bar{\theta}}{K}$ (coefficient to 2nd term)

We assume optically thick atmosphere $\Rightarrow$ most layers don't get flux from the ground i.e. for most layers, $e^{-\tau} \approx 0$

$t = e^{-(\tau - \tau')}$: note that t is negligible unless $\tau \rightarrow \tau'$, in which limit, its close to 1.

We get —

$$I_+(\tau) = \pi B(T(\tau)) - \pi B'(T(\tau)).\,T'(\tau)$$

Since $T'(\tau)$ is -ve, both terms are positive

$$I_-(\tau) = \pi B(T(\tau)) + \pi B'(T(\tau)).\,T'(\tau)$$

Second term is negative

First term is the flux emitted by the layer

Heating rate —

$$h_\nu = \frac{d}{d\tau}(-I_+ + I_-) = \frac{d}{d\tau}\left(2\pi.\, B'(T(\tau)).\,T'(\tau)\right)$$

$$h_\nu = \frac{d}{d\tau}\left(D.(\nu, T)\cdot\frac{dT}{d\tau}\right)$$

NOT near the boundaries

This is the std. eqn for diffusion. If we convert to pressure coordinates, the diffusivity D $(= 2\pi B'(T(\tau)))$ will turn out to be inversely proportion to specific absorptivity K. This means, optically dense pressure-layers tend to stay isothermal, while thinner layers allow more diffusion of heat

When τ increases, K increases $\Rightarrow$ D decreases. Thicker the optical thickness, harder for heat to diffuse

We can calculate back radiation − $I_-(0)$ ie. radiation from layers near it

We can think of an optically thick atmosphere as a set of isothermal slabs stacked vertically

⇉ Optically thin limit ⟶ NO BARRIER TO RADIATION IN TERMS OF ABSORPTION

Here, absorptivity of a pressure layer becomes additive & is equivalent to optical thickness

i.e. $e^{-\tau} \approx 1 - \tau$ where $\tau = \int_0^p \frac{d\tau}{dp} \cdot dp$ From Taylor Series

In this limit, for a layer τ, well above the surface,

$$I_+(\tau) = (1-\tau)\, I_+(0) + \int_0^{\tau} \pi B(T(\tau')) \cdot (1-(\tau'-\tau)) \cdot d\tau$$

where $\tau \approx \tau'$ as is very optically thin

$$\Rightarrow I_+(\tau) = (1-\tau)\, I_+(0) + \int_0^{\tau} \pi B(T(\tau')) \cdot d\tau'$$

$$I_-(\tau) = (1-(\tau_\infty - \tau))\, I_-(\tau_\infty) + \int_{\tau}^{\tau_\infty} \pi B(T(\tau')) \cdot d\tau'$$

Each layer has its own emissivity $d\tau'$ − we add it up and find the equivalent T that a body of τ_∞ would appear as a shell in our shell model

Like making the atm a single object in our Entire atm radiative balance eqn. But we haven't integrated over frequencies yet. $B(\bar{T}, \nu) = \frac{1}{\tau_\infty} \int_0^{\tau_\infty} B(\nu, T(\tau')) \cdot d\tau'$

If we write Kirchhoff's Law equations with the averaged-out radiation, we get −

$$I_+(\tau_\infty) = (1-\tau_\infty) \cdot I_+(0) + \tau_\infty \cdot \pi B(\bar{T})$$

$$I_-(0) = (1-\tau_\infty) \cdot I_-(\tau_\infty) + \tau_\infty\, \pi B(\bar{T})$$

T variations are quite small in such an atm − nearly isothermal

~~T̄~~ = An optically thin atm. acts precisely like an isothermal slab with temp $\bar{T}$, and small emissivity τ_∞. In the optically thin limit, bulk radiative effects of atmosphere mimics that of isothermal layer slab

Very less radiation is absorbed So heating rate also v. small

∵ small emissivity

Bulk emissivity − τ_∞ − v. less

Bulk temp − $\bar{T}$

Heating rate at this limit –

$$h_\nu = [I_+(0) + I_-(\infty)] - 2\pi B(T(\tau))$$

On converting to pressure coordinates, we must multiply by $\frac{d\tau}{dp}$, which is low by defn in optically thin limit. This means that heating rate is low due to radiative transfer.

If you want to heat atmosphere, at any ν, the atm shouldn't be optically too thick or thin. For efficient heating, $\tau_\infty \sim 1$ (order of magnitude)

---

* For ν where atm is opt. thick – relatively stable T profile. This eqn resembles eqn for diffusion – intuit it as heat diffusing along T gradient – no bulk flow of heat
* For ν where atm is opt thin – we get a stable (nearly isothermal) profile). Each layer is independent of others – each radiating at their own T.
* Boundary effluxes (OLR $I_+(\tau_\infty)$ and back radiation $I_-(0)$) can be calculated by looking at layers

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Lecture 11

Modelling Radiative transfer – Grey Gas model

Longwave vs Optical depth in Grey Atmosphere

Allows us to calculate which part of spectrum contributes most to heating

Grey gas – longwave

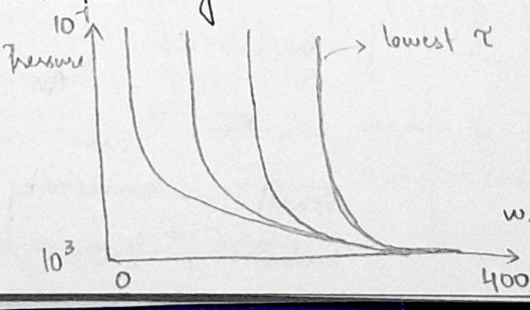
radiation_lw = climt.GrayLongwaveRadiation()

We're changing pressure at top to be 0.1 Pa

Air T is calculated using dry-lapse T profile

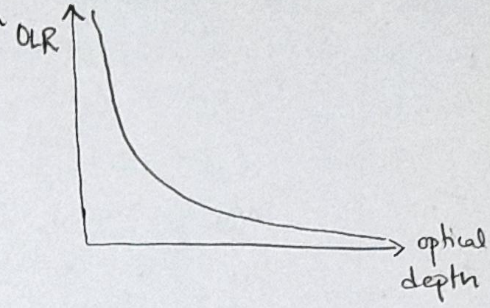
We explore the model for various τ_∞ values

Upwelling radiation decreases with height (almost exp)



All profiles become constant after a certain height

Variation of OLR with optical depth



Lots of exercises!

Think and discuss this?

Lecture 12

Optically thick / thin limits

Refer non-isothermal cases in pg. 25.

Optically thick ⇒ atmosphere where absorption due to greenhouse gases is so high that τ changes rapidly with height ⇒ $\tau_\infty \gg 1$

If $T_a \approx T_{surf}$ then there'll be radiative cooling of surface

if T decreases with temp. then there's radiative heating.

Lecture 13

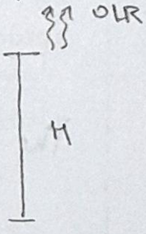
Grey gas radiative transfer – all in infrared spectrum (→ flux)

Simplified Schwarzschild equations : equal emissivity & absorptivity across all frequencies ⇒ no colors

$K = C$ $\quad K \neq K(\nu)$ $\quad$ if $K = C = 1$, then while optical thickness is independent of ν

Skin temperature $\quad \alpha$: absorption / emission coeff

Very little absorbed in the skin layer – if transmits (?) OLR. Low p layer



The only energy it receives is OLR, nothing from above

~~At~~ Input : $\alpha \cdot OLR = \sigma T_{rad}^4$

Output : $\alpha\, 2 \sigma T_{skin}^4$

$\Rightarrow T_{skin} = T_{rad}\, 2^{-1/4}$ ⇒ There's a big jump from T_{rad} to T_{skin}

$= 1.189$

Downwelling radiation is ignored

(30)

The radiating height is H. As optical thickness of atmosphere increases, H decreases until optically thin limit is reached when skin layer is just above surface

$\Rightarrow T_{skin} \simeq$ just above surface

$T_{rad} \simeq T_{ground}$

Then there'd be a big jump b/w ground & skin layer. This is a cool surface above a warm surface - not necessarily stable. For an optically thin limit, there's a discontinuity right above surface which moves higher as optical thickness increases.

Grey gas approximation

$$\int_{\nu} \frac{d}{d\tau} I_+ = \int_{\nu} -I_+ + \pi B(\nu, T(\tau))$$

We remove ν dependence of τ

WKT. $\int \pi B(T) = \sigma T^4$

Boundary flux for optically thin limit -

$$I_+(\tau_\infty) = (1-\tau_\infty) I_+(0) + \tau_\infty \pi B(\bar{T})$$

For $\tau_\infty << 1$

$$\begin{cases} I_+(\infty) = (1-\tau_\infty) I_+(0) + \tau_\infty \sigma \bar{T}^4 \\ I_-(0) = (1-\tau_\infty) I_-(\infty) + \tau_\infty \sigma \bar{T}^4 \end{cases}$$

In both eqns input from atmosphere is same - think of it like atm being a single blackbody slab, radiating equally up & down

Special limit - assume that only process setting T of planet is radiation (infrared)

The atmosphere at Radiative equilibrium will have a steady T profile - no more heating or cooling : independent of τ

$$h_\nu = \frac{d}{d\tau}(I_+ - I_-)$$

hence $h_\nu = 0$?

i.e. $I_+(\infty) = OLR$

Upwelling rad at τ_∞ = OLR

$$\bar{T}^4 = \frac{1}{\tau_\infty}\int_0^{\tau_\infty} T^4 \, d\tau$$ * This follows from assumption of infrared equilibrium

Applying upper boundary condition & taking difference b/w two equations of I_+ & I_- (?)

$$h_0 = 0^* = -\frac{d}{d\tau}(I_+ - I_-) = -(I_+ + I_-) + 2\sigma T^4$$

This gives us T in terms of $(I_+ + I_-)$

Taking sum of eqns for I_+ & I_- gives –

$$\frac{d}{d\tau}(I_+ + I_-) = -(I_+ - I_-) \qquad -(I_+ - I_-) = -OLR$$

$$\Rightarrow \boxed{2\sigma T_4 = (I_+ + I_-) = (1 + \tau_\infty - \tau)\, OLR} \quad — (\#)$$

This gives us pure radiative equilibrium temperature profile.

As $\tau \to \tau_\infty$, $T \to T_{skin\ layer}$ which means

If atm is optically thin, $\tau - \tau_\infty \to 0$ atm is ~~opti~~ isothermal, with $T \sim T_{skin}$

Since we know $(I_+ + I_-)$ and $(I_+ - I_-)$, we can estimate

$$I_-(0) = \frac{1}{2}\left[(I_+ + I_-) - (I_+ - I_-)\right] = \frac{1}{2}\left[(1+\tau_\infty)OLR - OLR\right]$$

$$\therefore \quad I_-(0) = \frac{1}{2}\tau_\infty \, OLR$$

In the optically thin limit, since $\tau_\infty \ll 1$, this means that there's very little backradiation ie most of the energy is lost to the space ie atmosphere doesn't play a big role in energy balance.

As τ_∞ increases, back radiation increases. In the optically thick limit, back radiation is much greater than OLR.

If we assume planet is in radiative equilibrium with absorbed radiation, $(1-\alpha)S$ where α: albedo, then $OLR = (1-\alpha)S$

This quantifies greenhouse effect – extra heat the ground has as compared to the air immediately above due to downwelling

Jump is small in opt thick & its a jump all the way to T_{skin} in optically thin atm

Then radiative energy budget –

$$\sigma T_g^4 = (1-\alpha)S + I_{\downarrow}(0) = \boxed{(1-\alpha)S\left(1+\tfrac{1}{2}\tau_\infty\right) = \sigma T^4}$$

$\Rightarrow$ As τ_∞ increases, T_g increases without bound

Low level air T : $\sigma T(0)^4 = (1-\alpha)S\left(\frac{1}{2}+\frac{1}{2}\tau_\infty\right)$

$\Uparrow$ From eqn (#) in Pg 29

$2\sigma T(0)^4 = (1+\tau_\infty)\cdot OLR$

$$\therefore \frac{T(0)}{T_g} = \left[\frac{\left(\frac{1}{2}+\frac{1}{2}\tau_\infty\right)}{\left(1+\frac{1}{2}\tau_\infty\right)}\right]^{1/4}$$

Hence, $T_g > T(0)$

$T_g \propto \tau_\infty$: Greenhouse gas

Gases cause global warming by increasing τ_∞

Through the above eqn, we can quantify the jump in temperature

Lecture 14.

Modelling Radiative Equilibrium

Evolution of atm T if radiation is the only process transferring energy.

Stratosphere is an example where

Slabsurface – homogenous surface of fixed density & specific heat which represents the surface of our model

\# Something abt tendencies_in_diagnostics = True

It's an interactive surface which absorbs incoming radiation and it radiates in infrared, and being which gets transferred through atm and it it to equilibrium.

Dynamic Energy balance is possible only if we have an interactive surface, not if we input T_g which would remain constant.

This model has 2 active components

Input : shortwave flux (200), layer thickness (1), air T (200)

Behavior of model becomes more realistic if we increase 'layer-thickness' but it'll also take a long time to run.

Excercises : Bunch of 'em
Try to think & answer

Lecture 15

Thermodynamics of dry air

There's a discontinuity in T profile in radiative equilibrium - thermodynamics helps us explore the stability of hot layer of air below a cool layer.

"Dry" thermodynamics - no condensation
Consider a parcel of air - as it rises up in air, if it encounter a temperature that shouldn't would make its constituent particles condense

$$dQ = dU - PdV \quad : \quad 1^{st} \text{ Law}$$

We use normalised version of 1st law where its divided entirely by mass

$$\Rightarrow \cancel{d}\ \delta Q = C_v dT + P d\rho^{-1}$$

To evaluate stability, we need to ~~contrast~~ compare 2 parcels of air at different T & P.
If $T_1 < T_2$ and $P_1 < P_2$ then it's not necessary that first air parcel will sink.

Measuring ρ is tricky - so we need a form of eq^n that depends on T and P.

$$P d\rho^{-1} = d(P.\rho^{-1}) - \rho^{-1} dP$$ substituting this in above eq^n

(34)

$$\delta Q = (c_v + R)\,dT - \rho^{-1}dp$$

RHS still can't be written as a perfect differential
⇒ it's path-dependent. Hence, δQ can't be used

$$ds \equiv \frac{\delta Q}{T} = c_p\frac{dT}{T} - R\frac{dp}{p} = \boxed{c_p\, d\ln\left(T p^{-R/c_p}\right) = ds}$$

Assuming c_p is constant, we now have a perfect differential.

Let's assume that air parcels move ~~adiabatically~~ vertically in an adiabatic fashion

$\delta Q = 0 \Rightarrow ds = 0$

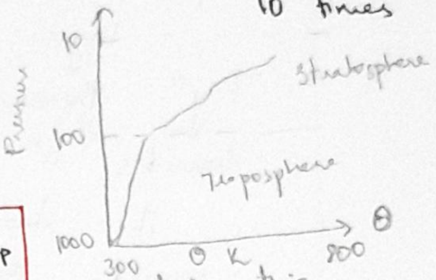
⇒ We (parcel) can move faster than any processes that removes energy from system

R/c_p Radiation time ≈ weeks ; Movement time is smaller by 10 times

$$c_p\, d\ln\left(T p^{-R/c_p}\right) = 0$$

$$\Rightarrow \ln\left(T p^{-R/c_p}\right) = \text{constant}$$

i.e. $\boxed{T_0 P_0^{-R/c_p} = T_f P_f^{-R/c_p}}$

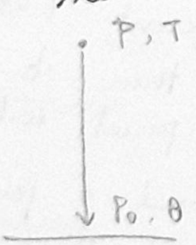


Maintain this ratio to remain const.

$$\theta = T\left(\frac{P}{P_0}\right)^{-\frac{R}{c_p}}$$

Pressure P_0 & T varies to remain const.

This allows ~~up~~ us to compare parcels at different heights ~~and~~ by considering the pressure and see which parcel rises and which sinks.



Say parcel is (P, T) is brought to P_0. It attains temp θ called potential temperature which tells us the potential of parcel to become warmer or cooler wrt some reference

$$\Gamma_p = \frac{d\theta}{dz}$$

Since p decreases with height, temp also (generally) decreases with height unless there's a source of energy - this decrease is known as Adiabatic Lapse Rate. $-\frac{dT}{dz}$

In stratosphere, slope of θ is high ⇒ air parcels are v. resistant to vertical displacement

Well-mixed atmosphere have a constant θ, not T ∴ adiabatic

$$\frac{\rho^{-1}dp}{T} = \frac{v}{\varepsilon_T}\frac{dp}{T} = \frac{R\,dp}{p}$$

$$\frac{d\ln T}{d\ln p} = R/c_p$$

Dry adiabat : $T(P) = \theta \left(\frac{P}{P_0}\right)^{+R/C_p}$ where $\theta = T(P_0)$

This T profile is called the Dry Adiabat.
An atm is stably satisfied ~~wi~~ if θ increases with height. If θ is the same with height, then its neutrally stable.
Vertical movement happens when atm is not stable and air parcels mix with each other.
For ideal gas, we can use Law of equipartition of energy to arrive at stuff.
Monotonic gas : 3 DOF $\Rightarrow \frac{3}{2}RT = C_v \Delta T$
Diatomic gas : $\approx$ 5 DOF (Rather than 6) Some DOF can be inaccessible at lower T. So gas behaves as if it has 5 DOF
This is important $\because$ T profile depends on C_p.
This will influence the lapse rate
$\Rightarrow$ As global T increases, more degrees of freedom will be accessible and this changes C_p.
All these calculations hold when gases are well mixed
But its possible that they're not well mixed
– eg. water vapour conc. varies with height and position on globe.
For any vol. of dry air, it's heavier than same vol. of water vapour. Hence, if you replace a portion of air parcel with water vapour, it will become lighter than its surroundings
Hydrostatic Relation : relating height with pressure
For a static fluid, this holds perfectly true –
$\therefore$ $A \cdot dP = -A\rho g dz$

A : area

$$\boxed{\frac{dP}{dz} = -\rho g}$$

This holds true for atmosphere when vertical acceleration of air is small compared to g $\Rightarrow$ no strong convection

This condition holds except in clouds - formation.

$$\delta Q = c_p dT - \rho^{-1} dp$$

$$\delta Q = c_p dT - \rho^{-1}(-\rho g dz)$$

$$\delta Q = c_p dT + g dz \Rightarrow \boxed{d(c_p T + gz) = DSE}$$

DSE : dry static energy

It gives the energy content in a column of the atmosphere

Since its a state variable, it can be computed through just boundary values. → (ΔDSE)

δ (DSE) = net input of energy into the column

Since entropy has dependence of T, this can't be computed easily. So, moist static energy is used to compute the energy budgets etc

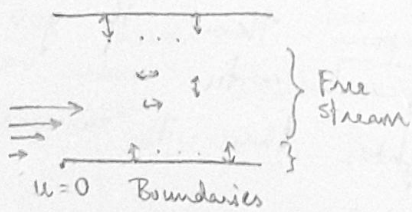
DSE More convenient than S

Lecture 16

Role of the Surface

The surface not only transfer energy through radiation - there are other processes through which energy can be transferred to atm.

Surface Boundary condition



Channel

Movement of particles near the boundaries is not simple - not linear

There's some friction - viscosity

speed of fluid decreases as it comes near the boundary

Viscosity is the 2nd order derivative we need

$$\rightarrow \frac{d}{dx}\left[\nu \frac{du}{dx}\right] = \nu \frac{d^2u}{dx^2}$$

ν: diffusion

ν → mol. viscosity

Viscosity removes momentum from the liquid - this is called drag

Drag takes away kinetic energy through friction & converts it to thermal energy.

Through experiments — drag is dependent on flow! It would suddenly become very high after a certain fluid velocity. Reynolds no. is used to characterize these two types of flow

RE measures the strength of flow $\left(\frac{u^2}{L}\right)$ and mol. viscosity (ν)

$$\boxed{Re = \frac{u^2}{\nu L}}$$

Small RE → laminar flow

Large RE → Turbulence

When turbulence sets in, the drag it feels is huge — true for all large-scale atmospheric phenomena. $Re \sim 10^5$

Momentum / Energy Fluxes.

Surface absorbs all radiation i.e $T_s > T_a$.

The processes that take away momentum from air (near the atmosphere) also transfer heat through diffusive processes.

Bulk Aerodynamic Formulae :

$$\text{Flux} = \underset{\text{density}}{\rho} \times \underset{\text{const.}}{c} \times \underset{\substack{\text{typical}\\ \text{velocity}}}{U} \text{ (grad. qty)}$$

For momentum, flux $= \rho . c . U (U - 0) = \rho c U^2$

temperature. flux $= \rho c U (T_s - T_a)$ Say, above a lake

water vapour flux $= \rho c U (q_s - q_a)$ q_s: saturated ~~vapor~~ specific humidity

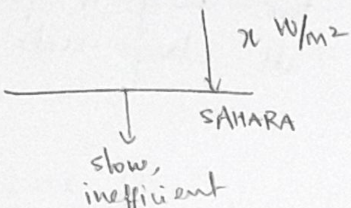
There 3 exchanges of momentum, temp. & water vapour are primary factors that govern the global momentum & energy budgets of atmosphere and surface (ocean or land) — so they're studied extensively

C_d : drag coefficient for neutral static stability
It depends on landscape & small-scale stuff, which in turn influences energy / mom. flux
This has to be measured experimentally.
C_d : Roughness length ?

Energy Partitioning

Sensible heat $- C_pT$: literal bulk transport of heat through air / water

Latent heat $- Lq$: Transport of heat through evaporable water vapour. Plays a big role in deciding what happens at surface



For bulk transfer of sensible heat to be efficient, there should be a huge T gradient ie surface should heat up a lot!

Now consider there's some water on the surface. Transfer of heat through water is much more efficient ∵ it's multiplied by L. So T_{surf} need not be as high. This partitioning of energy is very important.
This also has an implication of radiating temp and hence radiating height.

Plants — they lose water (from subsurface) through transpiration ⇒ energy partitioning is determined by transpiration rate — dependent on CO_2 conc, nutrients.
$CO_2 \uparrow \Rightarrow Lq \downarrow$

Other than albedo, plants can affect climate by altering surface properties.

It's imp. to study the properties of soil also

Optical depth — above this, no radiative cooling

Lecture 17

Modelling Radiative - Convective Eq

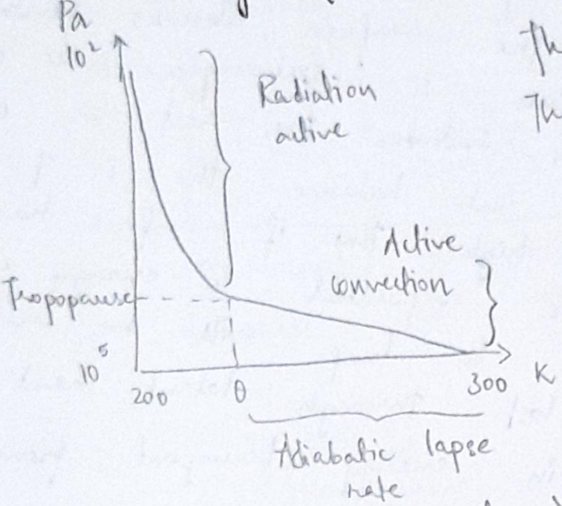
Dry Convective Adjustment — stable θ

Simple Physics — describes coupling b/w surface & atm layer through bulk aerodynamic formula

3 time steppers

There's a wind ∴ flux = $\rho c \underline{U}$ (...)

this makes sure there's flux — keeps boundary layer (a sink) 'active'.



There's a linear & exp curve

The point where it shifts is when radiation becomes dominant

θ value keeps moving up as convection tries to offset the instability caused by the radiation

Convection happens at very fast timescale compared to radiation.

The height / P at which the shift happens is tropopause — with troposphere below & stratosphere above. looking at OLR balance — convection offsets the difference between upwelling & downwelling radiation & adds its own flux to make it stable

Notice — very close to the surface, there's a anomaly in net OLR — its due to Rad + Conv + Boundary layers. Then till certain height (tropopause), its Rad + Conv, after which its just ~~to~~ radiative

(40)

Lecture 18

"Moist" thermodynamics

*partial pressures for mixture of gases

Atmosphere temp close to triple point of water,
This is the case most relevant to earth.

Condensible — a gas that can exist as a gas only upto certain P^* called Saturation vapour P.
i.e. intermolecular forces dominate & change the phase. to liq or solid

(?) This T is called critical point

There's a reduction in internal energy when gas → liquid called latent heat
This has profound implications on energy balance especially at surface

Without condensible, the surface absorbs shortwave energy from sun & exchanges some of it with atmosphere & radiates the rest as OLR
For this to come into balance, the T of surface should be v. high. But if surface has liquid, then it can be evaporated & energy can be transferred. So T_{surf} will be lesser
i.e energy is lost through latent heat.
This also helps in energy transport from tropics to the poles.
Using this method reduces the transfer by sensible heat

$$E = mC_pT + mLq$$ → fraction of vapours.

Humidity!!

If T is high, its harder for vapor to condense because ↑ KE. T and P (saturation vapour P) are linked — $P_s = f(T)$ P_{sat} – P at which condensation starts

ρ_c^{-1} : neglect $\quad \frac{d P_{sat}}{T} = \frac{1}{T} \frac{L}{(\rho_v^{-1} - \rho_c^{-1})}$ $\quad$ ρ_v : P of vapour, ρ_c : condensed phase

so we'll get eqn (#)

$$P_s = P_s(T_0)\, e^{-\frac{L}{R_a}\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

Saturation P is very sensitive to T

R_A : gas constant for vapour

$$\frac{d}{dT}(P_{sat}) = \frac{L}{R_A T^2} P_{sat}$$ As T falls, P_{sat} rises - fast

Single component atmosphere

How does the condensible affect temp. profile? Let's consider atmosphere has only 1 component - the condensible. Mars is a good example.

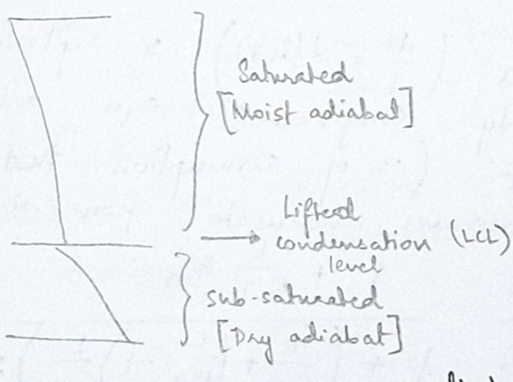
If its not condensible, we can derive dry adiabat.

As $P_{sat} \to P$, it reaches the temp where condensation takes place and energy is released which stabilises T. If entire atmosphere is saturated, T(p) is given by Clausius-Clapeyron Eqn

At T higher than T(sat), the atm is not saturated and at lower T, the gas condenses out

$$\boxed{T(p) = \frac{T_0}{1 - \frac{RT_0}{L}\ln\left(\frac{p}{p_s(T_0)}\right)}}$$: Moist adiabat

As P varies exponentially with T, as T decreases you quickly reach saturation



LCL: If you move a parcel of air mechanically upwards, then it condenses at this level

These 2 parts of atmosphere have different lapse rates. Dry adiabat is steeper than moist adiabat.

Because - in dry adiabat, only expansion causes cooling as its moved up. In the moist adiabat, as air parcel moves up, it expands and cools but at the same time, some moisture condenses and releases heat which warms up the air. This make T(p) less steep

Multi-component atmosphere

Mixture of condensible and non-cond gases. Non-cond gases are clumped together and weighted mean is used for calculations.

M_a : mass of non-condensibles

$$(M_a + m_c)\,\delta Q = M_a C_{pa}\,dT - \frac{M_a}{\rho_a}\,dP_a + m_c C_{pc}\,dT - \frac{m_c}{\rho_c}\,dP_c$$

$$+\ L\,dm_c \quad \text{- chemical potential}$$

When vapor condenses out, dm_c is -ve & L is positive

So '$L\,dm_c$' has a -ve sign $\Rightarrow$ it puts the system in a lower energy state i.e. heat is released

Usually, $\frac{m_c}{m_a} = r_c$ (mass mixing ratio) is used

$$\Rightarrow (1 + r_c)\frac{\delta Q}{T} = C_{pa}\frac{dT}{T} - \frac{dP_a}{\underbrace{\rho_a T}_{P_a}} + r_c C_{pc}\frac{dT}{T} - \frac{r_c\,dP_c}{\underbrace{\rho_c T}_{P_c}} + \frac{L\,dr_c}{T}$$

We assume that parcel is close to saturation

$\Rightarrow (r_c)_{sat} \approx r_c \qquad P_c \approx (P_c)_{sat}$

$$\boxed{\frac{m_c}{m_a} = r_c = \epsilon\,\frac{P_{c\,sat}}{P_a}}$$ where ϵ : ratio of mol weights

After more modification $\left(\frac{dT}{T} = d(\ln T)\right)$ & replacing r_c & all we get a fairly complicated eqn called the '<u>pseudo adiabat</u>' ($\because$ of assumption that any condensation removes condensate from air parcel).

$$\left\{\frac{d(\ln T)}{d(\ln P_a)} = \frac{R_a}{C_{pa}}\;\frac{1 + \frac{L}{R_a T}\,r_{sat}}{1 + \left(\frac{C_{pc}}{C_{pa}} + \left(\frac{L}{R_c T} - 1\right)\frac{L}{C_{pa} T}\right) r_{sat}}\right\}$$

This reduces to dry adiabat when $r_{sat} \to 0$

We saw that there are 3 ways of getting $T(P)$ –

1. Radiative equilibrium (for grey gas)
2. Turns out its not stable – there's vertical mixing of gases i.e. convection where θ (potential T) is constant with height

Where convection is dominant – that part of atmosphere is $\rightarrow$ TROPOSPHERE

The troposphere can be as shallow or deep as allowed by the atmosphere

Non-homogenity of stability – could happen when gases are not well mixed (eg. water vapour)

Water vapour is lighter than air. So if its added to an air parcel through evaporation, then that parcel can rise (not due to convection) because of change in composition

3. Condensible component of atmosphere ⇒ moist adiabat

No clouds in case of dry convection.

Clouds play an imp role – change albedo & optical nature

Fluid dynamics of moist adiabat is also v. different.

There are strong updrafts possible which can reach much higher heights than air parcels with higher potential T (θ)

Also, dry convection is local – just mixes with layer above

Moist convection is non-local – clouds rise and move. Hence tropopause is much higher in moist adiabat / convection than dry

Radiative or convective – usually convective equilibrium T(p) dominates because it acts at faster time scales